enough terms have been carried in the series, for if the individual terms are correct, the magnitude of the last group of three will be an error bound. A similar situation is found when calculating temperatures at other points.

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Flow in Pits of Fluid-Dynamic Origin

David T. Williams*
University of Florida, Gainesville, Fla.

A STUDY was made some years ago of the evidence of the nature of air flow over the surfaces of meteorites during their flight through air when they entered the earth's atmosphere. That evidence is left on the surface in the form of flow lines due to streaming of molten rock or iron oxide, just before the bodies in question were decelerated enough for ablation to cease.[1] In that study the suggestion was made, as based on the observations, that the shallow pits that have often been observed on meteorites, particularly iron ones, might have been caused by peculiarities of the air flow over the meteorite surface rather than by inhomogeneities of the meteorite material. In particular, it seemed likely that a pit would form in a surface if the flow that causes ablation can exist in the form of a "bound vortex," that is, a vortex with axis having a horseshoe shape which is held in one place on the ablating surface by the pressure gradients of the fluid flow. The pit would be formed at the apex of the horseshoe; the ends of the vortex would trail off to infinity in the wake. The horseshoe shape was concluded to be essential for the purpose of satisfying the continuity relation for the reversed flow implicit in any assumed vortex.

As a rather direct method for demonstrating that vortices of the type proposed actually exist over the surface of an ablating body and that, when they do, pits will be formed, tests were made by use of steady water flow over the surface of large salt blocks such as are fed to cattle. The relation between bound vortices and surface pits was verified in that pits were formed where bound vortices were predicted to exist, and not where they were absent. Figure 1 shows one of the pitted blocks. The evidence of the horseshoe shape of the bound vortex, presumed to have carved the pit, is clear in the shape of the lower one.

The question was posed how the ablation rate beneath a vortex could possibly be as great as that where no vortex exists; presumably, ablation rate depends on flow speed, which, in a vortex and especially inside a pit, certainly is less than it would be outside. An answer might be obtained if the flow about the pitted shape were studied in air flowing with the same Reynolds number as the water that shaped the block, under conditions of dynamic similarity.†

For measurement of the air flow inside the pit, a special velocity probe was made. It consisted of two lengths of 0.046-in. capillary tubing, each length closed at one end and pierced with a small hole in a side as near to the closed end as possible. The two tubes were soldered together with the holes in the sides carefully set to face outward with the axes colinear. When the probe is set normal to a stream of air, the pressure difference between the two tubes can be used to measure the air speed; in order to determine its flow direction, similar measurements can be devised.

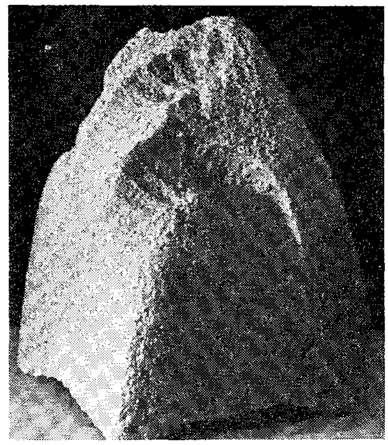

Fig. 1 Typical horseshoe - shaped pits caused by ablation. Rectangular 50-lb salt block subjected to erosion by moving water, 16 min at 4.8 knots.

The probe was used to map, in three dimensions, the flow of air within the lower pit in the salt cake of Fig. 1, under conditions of dynamic similarity to the flow that carved the pit. The results are shown in Figs. 2 and 3, selected from more complete data in the forementioned dissertation. The curves show the flow at two stations in the deepest part of the pit. Station 3 is a little to the right of the pit vortex, and station 4 is somewhat further down toward the right.

Figures 2 and 3 show tangential and axial components of the air flow velocity at each station, as observed in a traverse carefully chosen to be normal to the surface of the pit. The traverse also passes through a point where the velocity component is zero in the plane normal to the pit axis. The pit axis is defined as a line within the pit, at every point of which the tangential velocity component vanishes; it is, in brief, the vortex axis.

It is noted that, in each of Figs. 2 and 3, the tangential velocity component varies nearly linearly with distance from the vortex axis. The axial component is small at station 3; it is large at station 4 and clearly downstream in direction. The data clearly demonstrate the existence of a horseshoe-shaped vortex within the pit, as predicted on the basis of the pit shape and flow markings.

Of special interest is the observation that the maximum tangential speed of the air flow inside the pit is substantially less than that outside the pit; its magnitude is about half that of the flow near the salt cake surface where no pit exists.

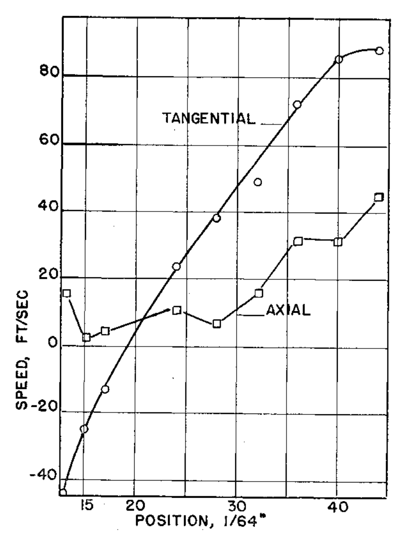

Fig. 2 Axial and tangential components of velocity of flow in the pit at station 3.

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* Professor of Aerospace Engineering.

† This note is a report of such a study carried out in a dissertation by William J. Larkin at the University of Florida.

In order to determine whether any reasonable mechanism exists for the fluid-dynamic formation of the pit, an attempt was made to measure the boundary-layer thickness at two points in the pit section at station 3. The measurement was carried out by use of the probe, with a hole drilled in the salt beneath it, in order to permit measurements very near the surface.

Figure 4 shows the results of the measurements, together with the results of a similar measurement at a point outside the pit. Apparent velocities different from zero were recorded below the surface of the salt, so that it is obvious that the presence of the hole renders the data meaningless at points very close to the surface. However, it was established in separate tests that the velocity distribution near the outer edge of a boundary layer is measured reliably by use of the technique of Fig. 4. The data were concluded to indicate that the boundary layer within the pit is thinner than that outside.

To be more explicit, the ablation rate at any point on an ablating surface may be expected to depend on the velocity gradient at the surface. This fact follows from the observation that the ablation rate is greater under higher-speed water flows, whereas at the surface itself the water speed is always the same, namely, zero. For making quantitative estimates of the slope, points were carefully located on curves 1 and 2, where the speed was 88% of the largest speeds observed, and the slopes of straight lines from those points to the origin were computed as estimates of the velocity gradients at the surface. The values obtained were 3760 and 3510 ft/in.-sec, respectively. The corresponding figure for curve 3 could not be computed, since the maximum speed was apparently at the salt surface; that is, the estimated velocity gradient would be very large.

If the estimated values of velocity gradient are significant, the ablation rate should be found to be greatest at station 3, rear, and least at station 3, center, with an intermediate rate outside the pit. The order is not at all the same as the magnitudes of the airspeeds outside the respective boundary layers; furthermore, the ratio of the estimated velocity gradients for curves 1 and 2 is 1.07, which is less than the ratio 1.79 of the maximum air speeds. That is, the boundary layers inside the pit are both thinner than that outside, according to this estimate. The estimated velocity gradients for curves 1 and 2 are, in fact, concluded to be the same, within the limits of uncertainty of the method of estimation employed. Therefore, the ablation rates also are estimated to be equal at the two points and greater for the point of curve 3.

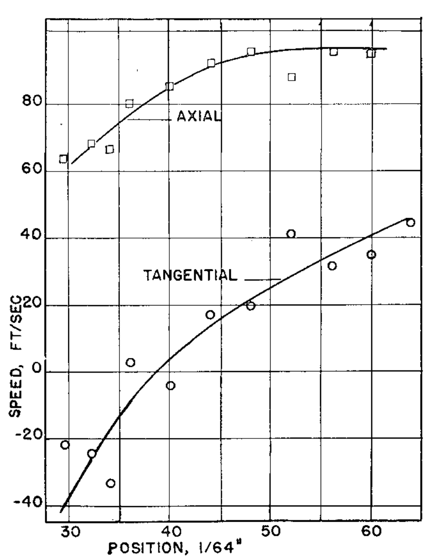

Fig. 3 Axial and tangential components of velocity of flow in the pit at station 4.

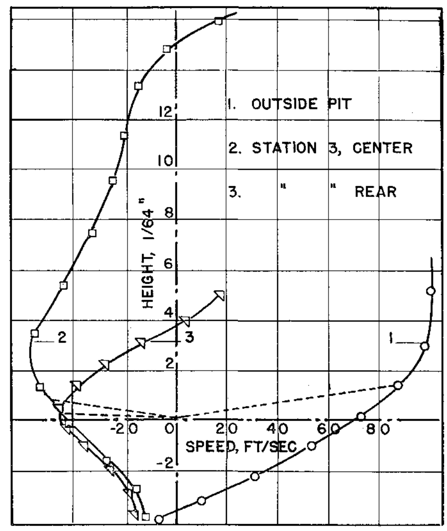

Fig. 4 A comparison of extrapolated velocity profiles (dotted lines) between center and rear edge locations in the pit at station 3, and a point outside the pit immediately upstream of station 3.

It is concluded that 1) measurements of the air flow within a pit, formed by water flow over a salt cake at equal Reynolds number, demonstrate the existence of the vortex flow that has been postulated as causing the formation of the pit; 2) longitudinal flow demonstrated in the vortex is observed to conform to the qualitative predictions of such flow in a stable vortex; 3) the maximum flow speeds inside pits is not high enough to account for pit formation in itself; and 4) measurements intended to determine boundary-layer thickness appear to justify the conclusion that ablation rates in a pit on a salt cake are as high as those outside, because the boundary layer is thinner in the pit as compared to the surface nearby, but outside, the pit.

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End Slopes of Column-Beams

JOHN W. HARVEY*
The Bendix Corporation, South Bend, Ind.

Nomenclature

c	= position ordinate—concentrated load
$j(i)$	= subscript denoting right (left) end
k	= index exponent
n	= series index
w	= load intensity
x	= position ordinate
y	= deflection ordinate
E	= modulus of elasticity
I	= moment of inertia
L	= length
P	= axial force
W	= transverse load
α	= ratio of axial force to critical load = PL^2/π^2EI
λ	= algebraic equivalent = $2L/EI\pi^2$
θ	= column-beam end slope
τ	= simple beam end slope
$\phi(\psi)$	= far (near) end slope magnification factor

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* Engineer, Analytical Services Department, Bendix Products Aerospace Division.